however, tends to recommend any scheme really competitive with it.

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A Discrete Search Procedure for the Minimization of Stiffened Cylindrical Shell Stability Equations

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Introduction

The analytical methods conventionally used for treating the stability of stiffened cylindrical shells under uniform axial compression and/or lateral pressure require the minimization of the buckling equations with respect to the number of axial half-waves (m) and circumferential full waves (n) where n and m are integers.[1–3] The usual procedures seem to be an exhaustive discrete search or a method combining an iterative minimization with respect to n with a discrete exhaustive search of m.[1,3] Timoshenko and Gere[4] discuss minimization methods for unstiffened and ring stiffened shells. Such procedures, although adequate where only a few designs are analyzed, would be inefficient if applied to shell synthesis, where many designs must be evaluated.[1,5–8]

This Note presents a more efficient, discrete, search method for treating this problem and includes a discussion of the nature of the buckling load functions for three lateral pressure loading conditions.

Minimization Problem

Consider the problem of finding the distributed unit axial compressive buckling load for a stiffened cylinder under uniform lateral pressure. For a given set of design parameters and pressure the buckling function $N_{cl}(n,m)$ has two discrete, independent variables, the axial and circumferential wave numbers m and n [see for example Eq. (31) of Ref. 2 and Eq. (29) of Ref. 3]. To obtain the buckling load, therefore, one must find the minimum of $N_{cl}(n,m)$ with respect to $n = 0,1,2,\ldots$, and $m = 1,2,3,\ldots$. This formulation can be viewed as an unconstrained discrete optimization problem and treated by an appropriate mathematical programming procedure.

The N_{cl} function can be represented by a surface in three-dimensional space if, for the purposes of illustration only, n and m are considered continuous. Contour maps of typical buckling load surfaces for the three possible pressure loading situations are shown in Figs. 1–3, where $N(n,m) = N_{cl}/N_{cl}^*$ and N_{cl}^* is the discrete minimum of N_{cl}. These are buckling values for a simply supported ring-stringer stiffened shell calculated using equations given by Burns.[1] It may be seen that in the absence of lateral pressure the surface is unimodal and a region of strong interaction exists between n and m with a resulting resolution ridge, starting at the $m = 1$ boundary. This ridge is encountered in most designs and can pose a serious problem for many search procedures. All surfaces of this type are, fortunately, unimodal.

Where there is significant internal pressure (Fig. 2), the unimodality remains but the resolution ridge typically vanishes. For the case of significant external pressure the surface is typically bimodal.[1] One of the local minima, the

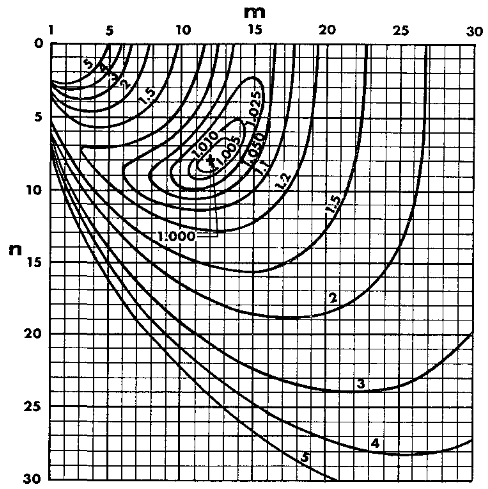

Fig. 1 Typical contour map of the $N(n, m)$ surface for a shell under no lateral pressure.

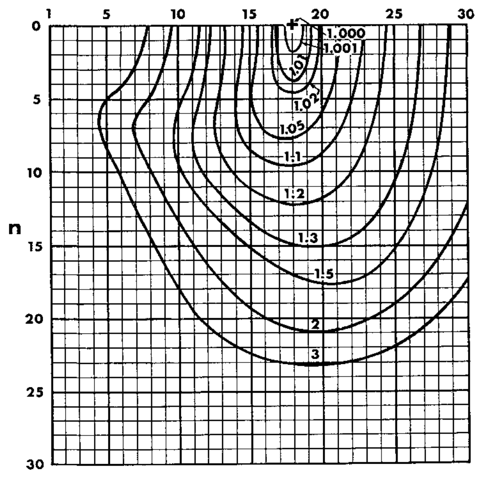

Fig. 2 Typical contour map of the $N(n, m)$ surface for a shell under significant internal pressure.

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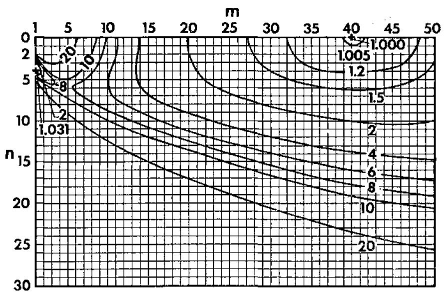

Fig. 3 Typical contour map of the $N(n,m)$ surface for a shell under significant external pressure.

"boundary minimum," always occurs on the $m = 1$ line. The second local "interior" minimum, although often on the $n = 0$ boundary, is frequently in the interior.

Minimization Procedure

A number of mathematical programming procedures can be applied to the aforementioned problem by treating n and m as continuous, locating the continuous minimum, and then checking the discrete points adjacent to this minimum. Better still, a lattice search procedure might be employed if the difficulties associated with the resolution ridges can be avoided. The particular scheme presented here is a modified, sequential, dichotomous search.[9] This search method was selected for four principal reasons in addition to its simplicity. First, the procedure is readily and efficiently modified to treat integer variables. The method is quite effective in locating minima where the interaction between variables is weak, such as in the surface represented by Fig. 2 or in the region away from the boundary minimum shown in Fig. 3. The dichotomous search can efficiently find the boundary minimum by searching the $m = 1$ line. Finally, the method is particularly well suited for use with automated synthesis procedures.

Starting from an arbitrary initial m^1 and n^1, a one-dimensional minimization is first performed with respect to m using a modified dichotomous search. For a fixed n the value of $N_{cl}(m^i + 1)$, $i = 1,2,3 \ldots$, is compared to $N_{cl}(m^i)$. If

$$N_{cl}(m^i + 1) \geq N_{cl}(m^i) \text{ and } m^i = m_a{}^i \tag{1}$$

then $N_{cl}(m^i)$ is the minimum. Here $m_a{}^i$ and $m_b{}^i$ define the lower and upper limits, respectively, of the search range. The initial values are $m_a{}^1 = 1$ and $m_b{}^1 = m_{\max}$ where $m_{\max}$ is the estimated largest value of m that could produce a minimum. In the event this estimate is small, and m converges to $m_{\max}$, the value of $m_{\max}$ is doubled and the search continued. When Eq. (1) is not satisfied, $N_{cl}(m^i - 1)$ is evaluated. Then, if

$$N_{cl}(m^i - 1) \geq N_{cl}(m^i) \leq N_{cl}(m^i + 1) \tag{2}$$

$N_{cl}(m^i)$ is the minimum. If Eq. (2) is not satisfied but if

$$N_{cl}(m^i - 1) \leq N_{cl}(m^i) \text{ and } m^i - 1 = m_a{}^i \tag{3}$$

then $N_{cl}(m^i - 1)$ is the minimum. When none of these three conditions are satisfied, the test point m^i and the search range are redefined such that

$$m^{i+1} = [(m_a{}^{i+1} + m_b{}^{i+1})/2]_T \tag{4}$$

$$m_a{}^{i+1} = m_a{}^i \text{ and } m_b{}^{i+1} = m^i \text{ if } N_{cl}(m^i + 1) \geq N_{cl}(m^i) \tag{5}$$

$$m_a{}^{i+1} = m^i \text{ and } m_b{}^{i+1} = m_b{}^i \text{ if } N_{cl}(m^i + 1) < N_{cl}(m^i) \tag{6}$$

where $[\ ]_T$ denotes truncations to the next lowest integer. The search is continued until the minimum is located.

The process is then repeated with respect to n for m fixed at the value for the minimum just located. The basic search is terminated when a set of search sequences fails to produce a change in the minimum n and m. This basic search procedure can, however, fail where the surface is rather flat or if the search is caught on the resolution ridge. A diagonal check is therefore performed at points of basic search termination if great accuracy is required or if the termination is along the $m = 1$ line. This diagonal check plus the basic search constitutes an exhaustive local search of all points adjacent to the minimum.

For most synthesis procedures, the redesign steps and, thus, the resulting changes in the buckling load surfaces are usually quite small. The entire search, therefore, often consists of only this local search, since n and m will frequently remain unchanged and the new search is started from the n and m of the previous minimum. This condition suggests another modification. An additional step can be taken in the direction of decreasing N_{cl}, if such a direction is found, at the start of a new search before initiating the sectioning procedure. Thus if there is only a unit change in the n or m of the new $N_{cl}{}^*$, this step, in addition to the local search, will locate the new minimum.

The ridge shown in Fig. 1 is troublesome for most search methods and in fact the procedure described here can occasionally fail on such a surface. Fortunately, however, an examination of several contour maps of this pressure loading condition indicated that if the search were started well away from the $m = 1$ line, it would always converge to the minimum.

For the case of the bimodal surface of Fig. 3, the global minimum can be found by comparing the local minima. The interior minimum is found by starting the search at an interior point well away from the boundaries (or starting from the previous interior minimum). The $m = 1$ line is then searched to find the boundary minimum.

Conclusion

The previous procedure was quite extensively and successfully applied to the automated design of stiffened shells described in Refs. 7 and 8. Some further improvement in search efficiency probably can be made but since the minimum is usually located during a brief local search the effect on over-all synthesis efficiency is likely to be quite small.

The form of the buckling equations can, however, have a major impact on over-all synthesis efficiency. Whenever possible the equations should be arranged in a form similar to Eq. (29) of Ref. 3 so that terms not involving n or m need be calculated only once during the $N_{cl}{}^*$ search procedure.

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